

B.Sc. Semester – 1 (NEP-2020) Examination

January-2024

Matrices and Coordinate Geometry (Major-1)

Time: 2:00 Hours

Marks: 50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any one.

- (1) Define: Periodic Matrix, Involutory Matrix, Symmetric Matrix, Orthogonal Matrix, Hermitian Matrix. (05)
- (2) Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and skew symmetric matrix. (05)

Que.1(B) Answer any one.

- (1) In usual notation for matrices A, B and C show that $A(B + C) = AB + AC$ (05)
- (2) In usual notation prove that $(AB)^T = B^T A^T$ (05)

Que.2(A) Answer any one.

- (1) Find Rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{bmatrix}$ (05)
- (2) Prove that the rank of the product of two matrices cannot exceed the rank of either matrix. (05)

Que.2(B) Answer any one.

- (1) Prove that the row rank of a matrix is equal to its column rank. (05)
- (2) Find Rank of matrix $B = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by represent it to normal form. (05)

Que.3(A) Answer any one.

- (1) State and prove Cayley – Hamilton theorem. (05)
- (2) Find Eigen values and Eigen vectors of matrix $C = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ (05)

Que.3(B) Answer any one.

- (1) Solve following linear equations. (05)

$$\begin{aligned} x + y + z &= 9 \\ 2x + 5y + 7z &= 52 \\ 2x + y + z &= 0 \end{aligned}$$
- (2) Find inverse of matrix $D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ using Cayley – Hamilton theorem. (05)

Que.4(A) Answer any one.

- (1) In usual notation derive the equation of a conic in polar co-ordinate $\frac{l}{r} = 1 + e \cos \theta$. (05)
- (2) Derive relation between Cartesian co-ordinates and polar co-ordinates. (05)

Que.4(B) Answer any one.

- (1) Derive the formula to find distance between two points in polar co-ordinates. (05)
- (2) Show that the sum of reciprocal of segments of any focal chord of conic $\frac{l}{r} = 1 + e \cos \theta$ is constant. (05)

Que.5(A) Answer any one.

(1) If A and B are given matrices such that $AB = BA$ then prove that (05)

$$(AB)^n = A^n B^n, n \in N.$$

(2) If $E = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find values of l, m , and n . (05)

Que.5(B) Answer any one.

(1) Find a and b such that the equations. (05)

$$x - 2y + 3z = 1$$

$$3x + y - z = 4$$

$$2x - 2y + az = b \text{ have (i) One solution (ii) No solution.}$$

(2) Find the polar co-ordinates for the Cartesian co-ordinates (1, 1) and the Cartesian (05)

co-ordinates for the polar co-ordinates $(1, \frac{\pi}{2})$
